The dependence of $\varepsilon_1^{ex}/\varepsilon_1$ on the parameters T_2/T_1, $A_1\varepsilon_2/\varepsilon_1A_2$, $A_1(1/A_2 - 1)(F_1/F_2)$, and $\varepsilon_1^{ex}(1/A_2 - 1)(F_1/F_2)$ is shown as a nomogram in Fig. 2. Each of these parameters is a quantity that is positive and greater than zero. It is clear from the nomogram (Fig. 2) that there is a complex dependence of $\varepsilon_1^{ex}/\varepsilon_1$ on the indicated parameters.

For $A_1\varepsilon_2/\varepsilon_1A_2 > 1$, just as for $A_1\varepsilon_2/\varepsilon_1A_2 < 1$, it is necessary to consider the cases $\varepsilon_1^{ex}/\varepsilon_1 < 1$ and $\varepsilon_1^{ex}/\varepsilon_1 > 1$. Thus, in the case $A_1\varepsilon_2/\varepsilon_1A_2 > 1$ when $\varepsilon_1^{ex}/\varepsilon_1 > 1$, the deviation of the experimental value from the actual value is smaller the closer to one the value of the ratio T_2/T_1, the greater than one the value of the quantity $A_1\varepsilon_2/\varepsilon_1A_2$, and the greater the quantity $A_1(1/A_2 - 1)(F_1/F_2)$ and the smaller the quantity $\varepsilon_1^{ex}(1/A_2 - 1)(F_1/F_2)$.

In the case $A_1\varepsilon_2/\varepsilon_1A_2 < 1$ when $\varepsilon_1^{ex}/\varepsilon_1 > 1$, the deviation of experimental value from the actual value decreases with a decrease in the quantities T_2/T_1 and $\varepsilon_1^{ex}(1/A_2 - 1)(F_1/F_2)$, an increase in the quantity $A_1(1/A_2 - 1)(F_1/F_2)$, and a trend toward one in the value of $A_1\varepsilon_2/\varepsilon_1A_2$; when $\varepsilon_1^{ex}/\varepsilon_1 < 1$, the deviation decreases with a trend toward zero for the value of $A_1\varepsilon_2/\varepsilon_1A_2$ and opposite changes in the other parameters.

Thus, an identical change of the same parameters does not have a unique effect on the accuracy of an experimental determination of emissivity, i.e., with realization of experimental apparatus by the two-cylinder method, one encounters considerable difficulties not only in the determination of possible deviation of the experimental value of emissivity from its actual value, but it is also extremely difficult to predict the most favorable conditions for performing an experiment in order to obtain the smallest deviation of the experimental value of emissivity from its actual value.

Only in the case $A_2 \to 1$ (black shell) do the quantities $\varepsilon_1^{ex}(1/A_2 - 1)(F_1/F_2)$ and $A_1(1/A_2 - 1)(F_1/F_2)$, and consequently the ratio F_1/F_2, have no effect and the ratio $\varepsilon_1^{ex}/\varepsilon_1$ is determined only by the values of the quantities $A_1\varepsilon_2/\varepsilon_1A_2$ and T_2/T_1. However, it is extremely difficult to predict a possible change in the quantity $A_1\varepsilon_2/\varepsilon_1A_2$. In such a case, the most advisable experimental conditions (for nongray bodies) must involve the realization of the smallest possible value for the ratio T_2/T_1.

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MODELING HETEROGENEOUS CHEMICAL REACTIONS UNDER THE ACTION OF A TURBULENT JET

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UDC 532.73-3

The chemical reaction of a solution with a metallic surface is modeled by the physical process of mass transfer. The general form of solution of the equation of convective diffusion is found and the possibility of calculating parameter values for which the process will occur efficiently is examined. The results of analysis are confirmed by experimental data.

It has been known from the time of Nernst that, when a metal dissolves in an aqueous solution, in most cases the process occurs in the diffusional region, where the effect of the molecular constant of the solution is negligible [1]. However, it has not been possible to reach a final conclusion as to the region in which heterogeneous reaction occurs when a metallic surface dissolves under the action of perpendicular turbulent jets of aqueous solution, since until recently this situation had not been studied from the appropriate point of view.

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In preparing electric circuits on foil dielectrics, in particular, copper surfaces are dissolved in perpendicular turbulent jets of solutions such as $CuCl_2$, $FeCl_3$, and $(NH_4)_2O_2S_8$.

This process can be efficiently carried out in a jet chamber, which contains closely spaced jets of solution moving uniformly in a direction perpendicular to the reaction surface. This surface is some distance $X/d_{eff} \geq 80$-100 from the output nozzle of an atomizer (d_{eff}) and lies within the developed turbulent boundary layer of a system of jets; the velocity v of the jets close to the reaction surface depends [2] on their velocity of emission U_0. In the main portion of the turbulent-jet system, the jets overlap to the extent of half their width in the directions of the length and width of the chamber, and, as a result, the velocity and concentration distributions in the directions parallel to the reaction surface (OY and OZ) are close to uniform, while the concentration gradient in the direction normal to the reaction surface (OX) is considerable.

There is considerable practical interest in understanding the kinetics of this process. Let us consider the case of a surface dissolving under the conditions indicated above, on the assumption of diffusional kinetics.

Since it has been arranged that the process is axisymmetric (involving a system of turbulent jets that are axisymmetric in the directions OY and OZ and uniformly overlapping), a two-dimensional investigation is possible. In this case, the mass-transfer equation in the turbulent region of an axisymmetric system of jets of aqueous solution moving perpendicularly to the reaction surface is

$$V_x \frac{\partial C}{\partial X} + V_y \frac{\partial C}{\partial Y} = \frac{1}{\mathrm{Pe}} \left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} \right). \tag{1}$$

Here $V_x = v_x/U_0$ and $V_y = v_y/U_0$ are dimensionless velocity components of the solution; $X = x/(b/2)$, $Y = y/(b/2)$ are dimensionless coordinates referred to half the width of the jets (b/2).

Analysis of Eq. (1) shows that increase in Pe decreases the right-hand side of the equation and the concentration distribution described by the equation is determined mainly by the left-hand side, corresponding to convective transfer. It is therefore of interest to find the values of the Peclet number used in practice to prepare electrical circuits in aqueous solutions.

When a metallic surface dissolves in jets of solutions of $CuCl_2$, $FeCl_3$, and $(NH_4)_2O_2S_8$ (for the following parameters of the process, obtained by experiment: $R = 17$ cm and $U_0 = 183$-798 cm/sec), the corresponding values of Pe are: $\mathrm{Pe}_{FeCl_3} = (0.4\text{-}2.0) \cdot 10^9$, $\mathrm{Pe}_{CuCl_2} = (0.5\text{-}2.0) \cdot 10^9$, and $\mathrm{Pe}_{(NH_4)_2O_2S_8} = (0.5\text{-}2.2) \cdot 10^7$.

Because the values of Pe for the aqueous solutions investigated are large, it is possible to assume that the reaction conforms to diffusional kinetics [1]. In the case under consideration — the turbulent boundary layer of axisymmetric flow at a surface — the process occurs by convective diffusion, and the considerable change in the state of motion in the direction OX allows the change in the mass transfer by molecular diffusion in the flow cross section to be neglected. Therefore (if $\partial^2 C/\partial X^2 \gg \partial^2 C/\partial Y^2$), Eq. (1) takes the form

$$V_x \frac{\partial C}{\partial X} + V_y \frac{\partial C}{\partial Y} = \frac{1}{\mathrm{Pe}} \frac{\partial^2 C}{\partial X^2}, \tag{2}$$

formally similar to the equation of mass transfer in the laminar boundary layer of a plate. But this purely formal similarity does not offer any possibility of finding an accurate solution of Eq. (2), because the flow picture in a turbulent flow characterized by the presence of velocity pulsations is unknown. In addition, whereas the boundary condition far from the reaction surface may be determined fairly simply ($C \to 1$ as $X \to \infty$), to find the boundary condition at the reaction surface involves establishing whether or not Stefan flow is present in the reaction. Consider a monomolecular irreversible reaction at a homogeneous metallic surface proceeding in solutions of $CuCl_2$, $FeCl_3$, and $(NH_4)_2O_2S_8$:

$$\left.\begin{aligned} &Cu + FeCl_3 - FeCl_2 - CuCl_2 = 0, \\ &Cu + CuCl_2 - 2CuCl = 0, \\ &Cu + (NH_4)_2\,O_2S_8 - CuSO_4 - (NH_4)_2\,SO_4 = 0. \end{aligned}\right\} \tag{3}$$

Analysis of these equations for reaction at a surface equally accessible to all the diffusing reagents shows that the boundary condition at the surface is characterized by the absence of Stefan flow and a zero value of the concentration C. In fact, in accordance with

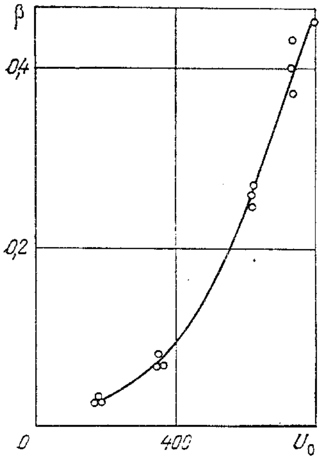

Fig. 1. Dependence of diffusion-rate constant β on velocity of emission U_0 of $CuCl_2$ jet perpendicular to copper surface; β, U_0, cm/sec.

Eq. (3), the change in volume in these reactions, determined by the magnitude and sign of the algebraic sum of the stoichiometric coefficients $\sum_i \nu_i$ of each of the reactions, is zero; this indicates that there is no additional flow of the reaction mixture to or from the reaction surface, while the velocity vanishes at the reaction surface. Therefore, the appropriate boundary conditions for the dimensionless equation of mass transfer in the main portion of the turbulent-jet system are

$$C \to 1 \text{ as } X \to \infty; \quad C = 0 \text{ for } X = 0. \tag{4}$$

By investigating Eq. (2) with the boundary conditions in Eq. (4) from the point of view of similarity theory, it is possible to find the general form of the solution (including an unknown function) and to establish which dimensionless parameters must be introduced into the desired solution:

$$C = F(X,\ Y,\ \mathrm{Pe}) = F(X,\ Y,\ \mathrm{Re\,Pr}) \text{ for } \frac{\beta}{U_0} = \text{const.} \tag{5}$$

Thus, the general form of the solution of the mass-transfer equation shows that the change in C depends on the material constant of the solution (Pr) and on the flow conditions (Re), and the same is true of the other physical quantities. As is known, probability theory gives no indication of the form of the function F, which, if analytic solution is difficult, may be determined experimentally. In particular, in terms of similarity theory, it is impossible to distinguish the powers of the dimensionless numbers appearing in Eq. (5). The powers to which Re and Pr are raised may be found by an experimental determination of the diffusional Nusselt number [$\mathrm{Nu} = (\beta \cdot R)/D$] as a function of Re and Pr

$$\mathrm{Nu} = \eta\, \mathrm{Re}^n\, \mathrm{Pr}^m \tag{6}$$

for known geometric conditions in the apparatus.

Various parameters for a metallic surface dissolving in turbulent jets of aqueous solutions of $FeCl_3$, $CuCl_2$, and $(NH_4)_2O_2S_8$ were investigated on sets of apparatus under identical or similar geometrical conditions (atomizer output nozzles of equal size, equal relative distances from the atomizer output nozzle, the same number of jets, etc.), making three or six repetitions for each measured amount of dissolved metal under each set of conditions [3].

The metallic surface was dissolved by a system of turbulent axisymmetric jets of aqueous solutions of $FeCl_3$, $CuCl_2$, and $(NH_4)_2O_2S_8$ emitted from the atomizer with $\mathrm{Re} = U_0 \cdot d_{\mathrm{eff}}/\nu \simeq 4 \cdot 10^4$. The value of U_0 was maintained constant (with an accuracy of ±1%) for all 12 jets of the apparatus and could be regulated. A transporter was used to produce uniform motion of the metallic plate under the jets of solution, with a velocity that could be adjusted in the range 0.2-0.9 cm/sec (smaller by a factor of 10^3 than the initial velocity of emission of the jets U_0). The distance of the dissolving surface from the atomizer output nozzles of direct-flow and centrifugal types was $X/d_{\mathrm{eff}} = 120$. In the apparatus, the jet system may be moved with respect to the dissolving surface at a velocity negligibly small in comparison with U_0. The thickness of the dissolving metal surface of the plate of foil dielectric was $50 \cdot 10^{-4}$ cm.

The effective solution-rate constant β is calculated by a procedure analogous to that in [1], taking into account the symmetry of the process:

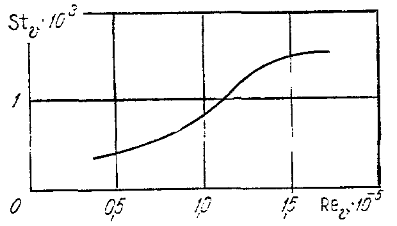

Fig. 2

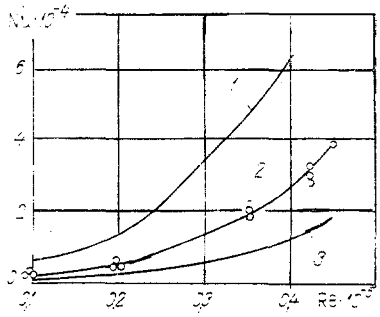

Fig. 3

Fig. 2. Dependence of Stanton number (St) on Reynolds number (Re) when copper dissolves in $CuCl_2$ jets; v = 183-290 cm/sec; b/2 = 6.0 cm; X/d_{eff} = 120.

Fig. 3. Dependence of the diffusional Nusselt number Nu on Re for copper dissolving in jets of $FeCl_3$ (1), $CuCl_2$ (2), and $(NH_4)_2O_2S_8$ (3).

$$\beta = \frac{-G_0 \dfrac{v}{U_0} \dfrac{A}{b/2}}{St/t_d} 2.3 \lg \frac{q_d - q}{q_d}, \quad \text{cm/sec.}$$

Here $G_0(v/U_0)$ is the volume flow rate of solution close to the reaction surface; q_d is the amount (g) of metal totally dissolved from a plate of area S = 64 cm^2; and q is the amount (g) of metal dissolved from the plate at time t.

Experimental results [3] on the effect of varying the velocity of jet emission U_0 show that the metal dissolves more rapidly as U_0 is increased (Fig. 1). This indicates once again that the reaction conforms to diffusional kinetics.

In addition, the form of the experimental dependence (Fig. 1) shows that under conditions of highly turbulent flow (for sufficiently high emission velocity of jets of solution) a nearly constant value of the Stanton number ($St = \beta/U_0$) would be expected. By analysis of the experimental curve in Fig. 1, the value of St can be found as a function of Re (Fig. 2): in the range $1.5 \cdot 10^5 \leq Re_v = (v \cdot R)/\nu \leq 1.74 \cdot 10^5$, St tends to a constant maximum value.

Functions of the form (6) may be found for various aqueous solutions by determining the index m for the external problem of flow (appropriate to the case of a metallic surface dissolving in jets of aqueous solution) from experimental data for various cases of heat transfer in forced flow. The value of m is determined experimentally over a wide range of Prandtl numbers (Pr = 0.7-1000) for the external problem of flow in heat-transfer processes and m = 0.31. The widely known theoretical basis for the method of analogy of diffusion and heat-transfer processes justifies the use of the value of m obtained in this way.

In addition, the value of m found from general theoretical considerations [1] for processes of solution in the turbulent region of flow agrees with the value of m for processes of heat transfer in forced flow. As a result, the dependence in Eq. (6) may be found for various aqueous solutions for similar geometric conditions on the basis of known experimental values of Nu and Pr (for Re = const) for one of them.

Curves of Nu = f(Re) calculated in this way for a metallic surface dissolving under the action of turbulent jets of aqueous $FeCl_3$ and $(NH_4)_2O_2S_8$ solutions are shown in Fig. 3, together with a curve of $Nu = f_{exp}(Re)$ obtained from experimental data for a metallic surface dissolving under the action of turbulent jets of $CuCl_2$. The results show that the reaction proceeds most efficiently in $FeCl_3$ jets; $CuCl_2$ jets are less efficient, and $(NH_4)_2O_2S_8$ jets are the poorest of all.

Comparing the conclusions of the present analysis with the experimental results in [3] shows that for the same residence time of the surface in $FeCl_3$, $CuCl_2$, and $(NH_4)_2O_2S_8$ jets, when the velocity of the solution close to the reaction surface is constant (v ≈ 71.5 cm/sec) and the geometric conditions are the same, the measured values of the solution-rate constants of the metallic surface are as follows: β_{FeCl_3} = 0.08-0.13 cm/sec, β_{CuCl_2} = 0.06-0.10 cm/sec, and $\beta_{(NH_4)_2O_2S_8}$ = 0.05-0.07 cm/sec; this confirms the results of the analysis.

When the Prandtl number of the solution falls below 150 (for Re = const and the geometric conditions determined by the symmetry of the process) the considerable reduction in the rate of the reaction may be compensated only by increasing the initial velocity of the jets of solution very considerably in comparison with that for solutions with $\Pr \geq 150$, which leads to engineering difficulties and is fairly uneconomic.

Thus, the general form of the solution of the mass-transfer equation for a metallic surface dissolving in turbulent jets of aqueous solutions and the analysis based on it are confirmed by experiment. This provides a basis for the choice of specific solutions such that the metallic surface dissolves most efficiently under the action of turbulent jets, and also the possibility of evaluating the suitability of new solutions.

NOTATION

ν_i, stoichiometric coefficients; β, diffusion-rate constant; D, diffusion coefficients; ν, kinematic viscosity of solution; d_{eff}, effective diameter of atomizer output nozzle; R, size (radius) of reaction surface; $V_x = v_x/U_0$, $V_y = v_y/U_0$, dimensionless velocity components of solution at reaction surface; U_0, initial velocity of jets on emission from atomizer; $C = c/C_0$, concentration at reaction surface, the ratio of the local value to the value C_0 far from the reaction surface; $X = x/(b/2)$, $Y = y/(b/2)$, dimensionless coordinates at reaction surface; b/2, half-width of jets of solution at reaction surface; $Nu = (\beta \cdot R)/D$, diffusional Nusselt number; $Pe = (U_0 \cdot R)/D$, diffusional Peclet number; $Pr = \nu/D$, Prandtl number; $Re = (U_0 \cdot d_{eff})/\nu$, $Re_v = (v \cdot R)/\nu$, Reynolds numbers referred to the emission velocity of the jets and their velocity at the reaction surface; $St_v = \beta/v$, Stanton number; G_0, flow rate of solution at atomizer outlet; A, half-amplitude of motion of atomizer with respect to the reaction surface; t/t_d, ratio of the residence time of the surface in the jets to the time at which the surface is completely dissolved.

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CALCULATION OF FLOW OF A MEDIUM IN THE GAP BETWEEN A ROTATING DISK AND A FIXED BOUNDARY WALL

M. I. Tsaplin

UDC 532.526

This paper examines the problem of determining the friction coefficients on the side surface of a rotating disk and a fixed boundary wall for different directions of the flowing medium.

Article [1] gave theoretical relations which can be used to calculate the swirl of a medium flowing in the gap between a rotating disk and a fixed boundary wall. In performing such calculations one must know the friction coefficients on the disk and the wall. It is usually assumed that these coefficients can be determined from known empirical relations for a rotating disk and an infinite space [2], if the Reynolds numbers for the disk and the wall are calculated using the relative velocity, i.e.,

$$\left.\begin{aligned} \xi &= 0.0535\,\mathrm{Re}^{-0.2} \quad \text{for} \quad \mathrm{Re} \geqslant 3.1\cdot 10^5, \\ \xi &= 1.234\,\mathrm{Re}^{-0.5} \quad \text{for} \quad \mathrm{Re} < 3.1\cdot 10^5, \end{aligned}\right\} \tag{1}$$

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